A CLASS OF CONVOLUTION TRANSFORMATIONS

Fadhel Al-Musallam, Vu Kim Tuan

Abstract

Integral transformations of the form

$$f(x) \to \int_0^\infty [k(|x-y|) - k(x+y)]\, f(y) dy, \quad x \in \mathbb{R}_+,$$

are studied on $L_p(\mathbb{R}_+)$, $1 \le p \le 2$. Watson's and Plancherel's type theorems are obtained.

Mathematics Subject Classification: 44A05, 44A35

Key Words and Phrases: Fourier integral transform, Fourier cosine and sine transforms, convolution integral transformation, unitary transformation

1. Introduction

We begin by recalling the definitions of the Fourier transform, Fourier cosine and Fourier sine transforms which we denote by F, F_c, and F_s, respectively ([1], [9]),

$$\begin{aligned}
(Ff)(x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(y)\, e^{-ixy}\, dy, && \text{if} \quad f \in L_1(\mathbb{R}),\\
(Ff)(x) &= \frac{1}{\sqrt{2\pi}} \frac{d}{dx} \int_{-\infty}^{\infty} f(y)\, \frac{e^{-ixy}-1}{-iy}\, dy, && \text{if} \quad f \in L_2(\mathbb{R}),
\end{aligned}$$

$$(F_c f)(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(y) \cos(xy) dy, \quad (F_s f)(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(y) \sin(xy) dy,$$

if $f \in L_1(\mathbb{R}_+)$, and